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AD 634356

Office of Naval Research Contract Nonr-610(06)
Task Order NR 064-476

Technical Report No. 1

ON THE WESTERGAARD METHOD OF CRACK ANALYSIS

by
G. C. Sih

March 1966

Department of Applied Mechanics
Lehigh University, Bethlehem, Pennsylvania

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Lehigh University
Bethlehem, Pennsylvania

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On the Westergaard Method of Crack Analysis[1]

by

G. C. Sih [2]

A survey of the literature on the analysis of crack problems shows that the Westergaard method [1][3] has been most frequently quoted and used by the practitioners in fracture mechanics for nearly thirty years. Surprisingly enough it has yet to be pointed out that the method in [1] suffers severe restrictions for a class of problems dealing with the infinite medium with a crack (or cracks) subjected to external loads at infinity. These restrictions will be derived in the work to follow from the more general consideration of complex potentials originated by Muskelishvili [2].

In the theory of two-dimensional isotropic elasticity, the stresses and displacements may be expressed in terms of two complex functions $\phi(z)$ and $\psi(z)$ of the variable $z=x+iy$. They are

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1 The results presented in this paper were obtained in the course of an investigation carried out under Contract Nonr-610(06) with the Office of Naval Research in Washington, D.C.

2 Professor of Mechanics, Lehigh University, Bethlehem, Pa.

3 Numbers in brackets designate References at end of Note.

$$\sigma_x + \sigma_y = 4 \operatorname{Re} [\phi'(z)]$$

$$\sigma_y - \sigma_x + 2i \tau_{xy} = 2[\bar{z} \phi''(z)+\psi'(z)] \tag{1}$$

and

$$2\mu(u+iv) = \kappa \phi(z) - z \overline{\phi'(z)} - \overline{\psi(z)} \tag{2}$$

where κ takes the value $3-4\nu$ for plane strain and $(3-\nu)/(1+\nu)$ for generalized plane stress and ν is the Poisson's ratio. The shear modulus is denoted by μ. Eqs. (1) and (2) may be simplified by introducing symmetry conditions as follows:

<u>Symmetric Problems</u>. If the external loads are placed symmetrically with respect to the x-axis along which the cracks are situated, then the shearing stress τ_{xy} must vanish at $y=0$, i.e.,

$$\operatorname{Im} [\bar{z} \phi''(z)+\psi'(z)] = 0 \quad , \qquad \text{for} \quad y = 0 \tag{3}$$

Eq. (3) can be satisfied by taking

$$\psi'(z) + z \phi''(z) + A = 0 \tag{4}$$

where A is a real constant depending upon the applied load. Making use of eq. (4) and letting $\kappa = 3 - 4\nu$ for plane strain, the stresses take the form

$$
\begin{aligned}
\sigma_x &= 2\,\mathrm{Re}\,[\phi'(z)] - 2y\,\mathrm{Im}\,[\phi''(z)] + A \\
\sigma_y &= 2\,\mathrm{Re}\,[\phi'(z)] + 2y\,\mathrm{Im}\,[\phi''(z)] - A \qquad (7) \\
\tau_{xy} &= -2y\,\mathrm{Re}\,[\phi''(z)]
\end{aligned}
$$

and the displacements are

$$
\begin{aligned}
2\mu\, u &= 2(1-2\nu)\,\mathrm{Re}\,[\phi(z)] - 2y\,\mathrm{Im}\,[\phi'(z)] + Ax \\
& \qquad (8) \\
2\mu\, v &= 4(1-\nu)\,\mathrm{Im}\,[\phi(z)] - 2y\,\mathrm{Re}\,[\phi'(z)] - Ay
\end{aligned}
$$

Hence, the problem is reduced to the determination of a single complex function $\phi(z)$ satisfying the necessary boundary conditions. Eqs. (7) and (8) agree with eqs. (4-6) and (9-10) in [1], respectively, only if

$$
2\,\phi'(z) = Z\ , \quad A = 0
$$

In general, the constant A cannot be neglected arbitrarily. To illustrate this point, consider the problem of an infinite medium with a central crack of length 2a along the x-axis. The boundary conditions are

$$
\begin{aligned}
&\sigma_y = \tau_{xy} = 0\ , \quad y = 0\ , \quad -a<x<a \\
& \qquad (9) \\
&\sigma_x = \varepsilon\sigma^{\infty},\ \sigma_y = \sigma^{\infty},\ \tau_{xy} = 0,\ \text{as}\ (x^2+y^2)^{1/2}\to\infty
\end{aligned}
$$

The solution to this problem is given by [2]

$$\begin{aligned} \phi'(z) &= (\sigma^{\infty}/2)[z/(z^2-a^2)^{1/2}]-(1-\varepsilon)(\sigma^{\infty}/4) \\ \psi'(z) &= (a^2\sigma^{\infty}/2)[z/(z^2-a^2)^{3/2}]+(1-\varepsilon)(\sigma^{\infty}/2) \end{aligned} \tag{10}$$

Inserting eq. (10) into (4), A is found to be

$$A = -(1-\varepsilon)(\sigma^{\infty}/2)$$

Note that A vanishes only in the special case of $\varepsilon = 1$ corresponding to the case of uniform tension at infinity. The same applies to the problem of an infinite row of collinear cracks spaced periodically in an infinite medium [3].

<u>Skew-Symmetric Problems</u>. For loads applied skew-symmetrically with respect to the crack line, say along the x-axis, the normal stress σ_y is required to vanish at $y=0$, or

$$\mathrm{Re}\,[2\phi'(z)+\bar{z}\,\phi''(z)+\psi'(z)] = 0, \text{ at } y = 0 \tag{11}$$

It follows that

$$\psi'(z) + 2\,\phi'(z) + z\,\phi''(z) + iB = 0 \tag{12}$$

where B is a real constant. Substituting eq. (12) into eqs. (1) and (2) and separating the real and imaginary parts give

$$\begin{aligned} \sigma_x &= 4\,\mathrm{Re}\,[\phi'(z)] - 2y\,\mathrm{Im}\,[\phi''(z)] \\ \sigma_y &= 2y\,\mathrm{Im}\,[\phi''(z)] \end{aligned} \tag{13}$$

$$\tau_{xy} = -2\,\mathrm{Im}\,[\phi'(z) - 2y\,\mathrm{Re}\,[\phi''(z)] - B \tag{13}$$

and

$$\begin{aligned} 2\mu\, u &= 4(1-\nu)\,\mathrm{Re}\,[\phi(z)] - 2y\,\mathrm{Im}\,[\phi'(z)] - By \\ 2\mu\, v &= 2(1-2\nu)\,\mathrm{Im}\,[\phi(z)] - 2y\,\mathrm{Re}\,[\phi'(z)] - Bx \end{aligned} \tag{14}$$

The Westergaard version of eqs. (13) and (14) may be obtained by selecting an Airy stress function of the form $y\,\mathrm{Im}\,Z$. The results are the same as those given above if

$$2\,\phi'(z) = Z, \quad B = 0$$

The restriction of $B=0$ leads to a trivial solution for the problem of uniform in-plane shear applied to an infinite medium containing a crack. For this problem, the conditions are

$$\begin{aligned} &\sigma_y = \tau_{xy} = 0\,, \quad y = 0\,, \quad -a<x<a \\ &\sigma_x = \sigma_y = 0\,. \quad \tau_{xy} = \tau^\infty\,, \quad \text{as } (x^2+y^2)^{1/2}\to\infty \end{aligned} \tag{15}$$

From [2], the complex functions are

$$\begin{aligned} \phi'(z) &= -(i\tau^\infty/2)[z/(z^2-a^2)^{1/2}] + i\,\tau^\infty/2 \\ \psi'(z) &= i\tau^\infty[z/(z^2-a^2)^{1/2}] - i(a^2\tau^\infty/2)[z/(z^2-a^2)^{3/2}] \end{aligned} \tag{16}$$

The constant B may thus be found from eqs. (12) as

$$B = -\tau^{\infty}$$

Hence, B cannot vanish for a non-trivial solution.

It should be mentioned that the Westergaard method is valid for loads applied to the crack surfaces since in such cases the constants A and B have no contribution.

References

1. H.M. Westergaard, "Bearing Pressure and Cracks", Journal of Applied Mechanics, Vol. 6, 1937, pp. A 49-53.

2. N.I. Muskhelishvili, "Some Basic Problems of Mathematical Theory of Elasticity", P. Noordhoff Ltd., Groningen, Holland, 1953.

3. Private communication with I. N. Sneddon.

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4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

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Sih, G. C.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 1966	7	3

8a. CONTRACT OR GRANT NO. Nonr-610(06)	9a. ORIGINATOR'S REPORT NUMBER(S) No. 1
b. PROJECT NO. NR-064-476	
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13. ABSTRACT

The Westergaard method of crack analysis, published almost thirty years ago, is shown to be invalid for a class of crack problems dealing with the infinite medium with cracks under applied loads at infinity. The necessary modifications of the Westergaard method are derived from the complex potential formulation of Muskhelishvili. The examples of a single line crack in an infinite plate owing to biaxial tension and pure shear are discussed.

DD FORM 1 JAN 64 1473

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14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Fracture mechanics						
Cracks						
Plane extension						

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